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A CALIBRATION METHOD FOR STANDARD LINEAR-POLARIZATION ANTENNAS

A THESIS

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SUMMARY

Most measurements of the polarization characteristics of microwave antennas require the use of a standard linearly polarized comparison antenna. Commonly a standard gain horn is used as the linear polarization standard and the accuracy of the results therefore depends on the precision with which the standard is calibrated. This thesis presents a method for calibration of the standard antenna.

An analysis of the power transfer between two arbitrarily polarized antennas shows that a zero null (no power transfer) between two such antennas is possible only if the respective polarization characteristics of the two antennas are conjugate to each other. The term "conjugate" as used here means that the polarization ellipses of the two antennas, defined with respect to a common coordinate system, have inverse ratios of x to y components and opposite senses of rotation.

Based on the mathematical study, a trial and error procedure is presented in which three similar horn antennas are alternately compared and adjusted until all three can be shown to be purely linearly polarized. A second comparison procedure is described which enables the specification of the true direction of linear polarization.

An experimental verification of the procedure is presented in which the adjustment of the polarization characteristics of the antennas is accomplished through the means of distorting the input waveguide with a clamping pressure applied diagonally across the waveguide sections. Alternate procedures for adjusting the characteristics are desirable

and should be investigated.

A discussion of some of the pertinent aspects of the experimental work is presented. In particular, the effects of system noise level and antenna range effects are discussed with respect to their roles in limiting the accuracy of the final calibration.

In theory, the procedure presented here should enable one to calibrate a linear standard in which no cross components of polarization exist. From a practical standpoint, it is felt that ratios of primary polarization to cross polarization of 65 to 70 db are feasible.

CHAPTER I

INTRODUCTION

In measuring the polarization characteristics of medium to high gain microwave antennas, a difficulty arises which is common to virtually all measurement problems--that of acquiring a sufficiently accurate standard for the quantity being measured. All resistance measurements, for example, are basically comparisons between unknown resistances and standard resistances. This problem, of course, has received considerable attention and standard resistors are available for almost any required degree of accuracy.

Antenna polarization measurements generally require the use of a "purely linearly polarized" reference antenna as the measuring device. The purity of polarization of such devices has received little attention. Standard gain horns, for example, are widely used as linearly polarized reference antennas, but generally are presumed to have secondary components of polarization in the far field which are of the order of 40 db below the primary components. For many practical situations, such as measuring the circularity of a circularly polarized field, this level of secondary polarization components is too small to be of any consequence. However, when attempting to measure the linearity of a linearly polarized field, these secondary components of polarization in the test antenna can create gross errors.

Suppose as an example, a linearly polarized parabolic antenna is to be tested. Such antennas have cross components of polarization on the order of 30 to 35 db below the parallel component. Assume a typical value of 33 db for purposes of this example. A typical procedure is to orient the antenna under test as a receiver with its direction of primary polarization coinciding with some arbitrary space axis. The test antenna is then used as a transmitter and oriented to transmit in the same polarization as the parabola. After a convenient level is recorded on the recording equipment, the transmitter is rotated 90° about the boresight axis and the relative power of the cross component received by the parabola is recorded. If, as assumed, the parabola has a cross component at -33 db, the voltage at the detector is .022 times the original value. However, since the transmitter polarization is not pure, it is transmitting a cross component which is typically 40 db below the primary component. This component is now oriented in the proper polarization to be received by the parabola, hence the detector receives a second voltage which is .01 times the original value. Depending on the relative phasing of this error voltage, the detector can read any value from 29.9 db to 38.4 db which is in general an unacceptable degree of accuracy.

In addition, given a purely linearly polarized reference, there is no way of orienting the direction of polarization to coincide with a predetermined space axis. Conventionally, if a vertically polarized wave is desired from a horn antenna, one usually levels the broad dimension of the waveguide input section. However, there is no guarantee that this surface is exactly perpendicular to the true direction of polarization of the radiated field. Consequently, some method of determining the angular relationship between the true direction of polarization and a reference surface on the antenna is required.

The motivation for this investigation arose out of efforts to measure low level cross polarized components of the far field radiation patterns of several X-band radar antennas. In the light of the above discussion, the difficulty of making these measurements can be readily appreciated.

It is the purpose of this research to develop and describe a method of removing the low level cross polarized components from the far field radiation pattern of a simple electromagnetic horn. A second objective, after achieving a pure linear polarization, is to describe a method of orienting the direction of polarization to coincide with an arbitrary space axis to a degree of accuracy limited only by the available means of mechanical alignment.

A review of the literature on the subject of antenna polarization measurements reveals that apparently there has been no effort to resolve the problem of linear standards. Characterization of general elliptically polarized fields is well treated in the literature (1)-(4), and several authors have reported various methods of measuring antenna polarization characteristics (1), (5), (6). Most of the reported methods require the use of a linearly polarized test reference antenna, but none of the authors acknowledge the difficulty of obtaining such a test antenna. Kraus (1) recognizes the need for such a device and suggests the use of a dipole whose far field radiation pattern is well known to contain only a single component of polarization. However, the lack of directivity in the dipole radiation pattern creates the additional problem of reflections from surrounding objects which tend to disrupt any type of antenna measurement. Clayton and Hollis (6) also acknowledge the need for a pure test antenna, but offer no suggestions as to how to obtain one.

CHAPTER II

THEORETICAL DISCUSSION

A study of the mathematics of power transfer between two arbitrarily polarized antennas has been carried out and is presented in detail in the Appendix. Only those results which are pertinent to the experimental procedure will be presented here.

It can be shown that the general elliptically polarized wave can be completely described by specifying the relative magnitudes of two linear orthogonal components and the relative phase angle between them. It can further be shown that a linearly polarized wave can be achieved by either reducing one component of the elliptical wave to zero or by reducing the phase angle between the two linear components to zero.

Of specific interest to this work is the determination of the conditions imposed on the polarization characteristics of two antennas in a transmission system such that no power is transferred between them. The mathematical analysis shows that when the boresight axes of two antennas are oriented to be colinear and opposite in sense (aimed at each other), then the amount of power which is coupled between them is a function of the polarization characteristics of the two antennas. In particular, to obtain a zero null (no power transfer) between the two, it is necessary that the polarization characteristics be conjugate to each other. The term "conjugate" as used here means that the respective polarization ellipses of the two antennas, defined with respect to a common coordinate system, have inverse ratios of x to y components and opposite senses of rotation. In terms of the components of the field, the ratio of the two linear components of one antenna is the inverse of the ratio of the two linear components of the other, and the phase angle between the components of one is the negative of the phase angle between the components of the other. Obviously, the situation when both antennas are linearly polarized is a special case of conjugate polarization and occurs when one of the linear components is zero, or the phase angle is zero. In addition to being a special case of conjugate polarizations, the situation of both antennas being linearly polarized is also a special case of identical polarizations. That is, the polarization ellipses of the respective antennas have the same ratio of x to y components and the same sense of rotation. However, a zero null in the transfer of power between two antennas with identical polarizations is not possible unless both antennas are purely linearly polarized.

The above suggests a means of calibrating a linearly polarized standard. If two antennas are set up as described, and a zero null can be obtained while insuring that both antennas have identical polarization characteristics, then the two antennas must be linearly polarized.

The two antennas can be made to have identical polarization characteristics by introducing a third comparison antenna. If the two test antennas are independently nulled against the comparison antenna, then each must individually have polarization characteristics which are conjugate to the third antenna; therefore, the polarization characteristics of the two test antennas must be identical. If these two identical antennas are oriented as above, and one of them is rotated about its boresight axis while monitoring the power transfer, then the ratio of maximum response to minimum response will give an indication of the relative magnitude of the orthogonal linear components of the two antennas. In particular, if the minimum response is a zero null, then the ratio of maximum to minimum components of both antennas is infinite. That is, the smaller of the linear orthogonal components of the general elliptical wave is zero. Therefore, both antennas are purely linearly polarized.

Chapter IV will be concerned with describing an experimental procedure whereby two antennas can be made to have purely linearly polarized far field radiation characteristics. A trial and error method for achieving a zero null of power transfer between two antennas with identical polarization characteristics is outlined.

CHAPTER III

INSTRUMENTATION AND EQUIPMENT

Performance of the experiment requires the use of three similar electromagnetic horns constructed to be as nearly equal as possible. The horns are dual polarized and were constructed by flaring a square waveguide equally in both planes to form a square aperture. The square waveguide throat section has inside dimensions of .840 inch by .840 inch which will support propagation of the TE_{10} and TE_{01} modes to the exclusion of higher modes at a frequency of 9375 Mc. The horn is linearly tapered to an aperture of six inches by six inches.

Each horn is mounted on a set of bearings which allow a complete 360° revolution about the boresight axis. The horns are fed through a rectangular to square transition which allows them to be connected to conventional RG/52 waveguide. Each horn is equipped with a special clamping mechanism which applies a force diagonally across the square waveguide section enabling distortion of the waveguide to change the polarization characteristics of the horn. A photograph of one of the horn assemblies is shown in Figure 1. The cross hairs on the aperture provide a means of optically aligning the boresight axis of the horn.

Additional equipment required are a klystron source and power supply, a ferrite isolator, a pad attenuator, a tunable detector mount, and a suitable meter.

Since the experimental work was, in essence, an antenna measurement problem, it was necessary to choose a location which offered a

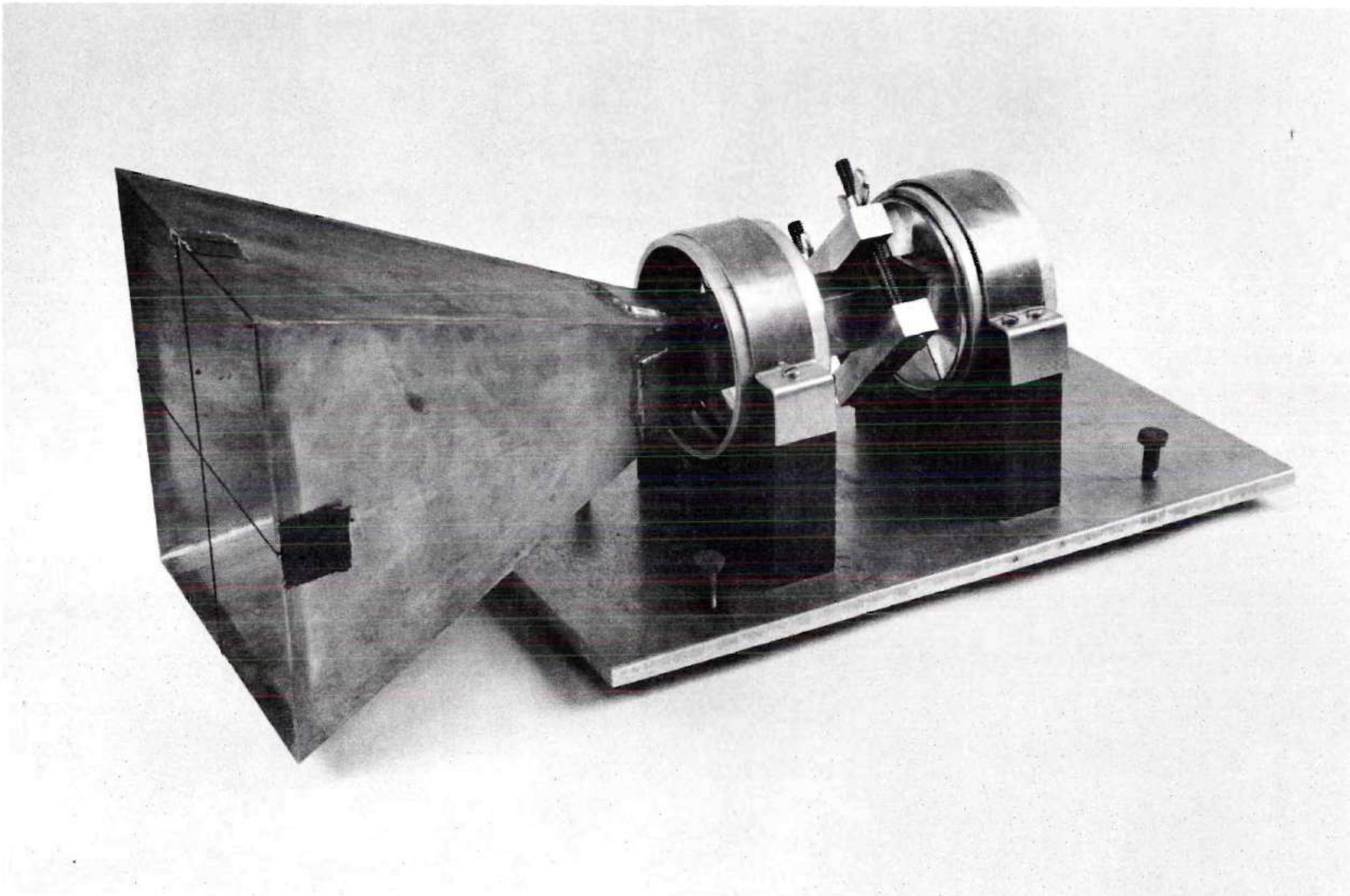


Figure 1. Photograph of one of the Antenna Assemblies.

reasonably good antenna range relatively free of interfering reflections. The microwave dark room of the Electrical Engineering Building was chosen as the most acceptable location.

CHAPTER IV

PROCEDURE

The experimental set-up of the equipment is depicted schematically in Figure 2. Only two of the three antennas are in use at a given time. A wavemeter can be inserted into the transmission line to tune the klystron to the operating frequency of 9375 Mc. For convenience the three antennas are arbitrarily designated A, B, and C.

Calibration of the Linear Polarization

Two of the three antennas are selected and set up in the system of Figure 2. In the interest of definiteness, let antenna A be the transmitter and antenna B be the receiver. The respective boresight axes of the two antennas are aligned to be colinear. This is accomplished optically with the aid of the cross hairs on the antenna apertures. A small plug with a peephole in the center is inserted in the square waveguide input of the antennas. By sighting through the peephole and aligning the cross hairs of both antennas it is possible to align the boresight axes.

The direction of principal polarization of the transmitter is arbitrary, but for convenience the antenna should be oriented such that the field is approximately "vertically polarized." A certain ambiguity exists here since the use of the term "vertically polarized" implies that the field is purely linear. Since the field of the horn at this time is, in general, elliptically polarized, the term "vertically

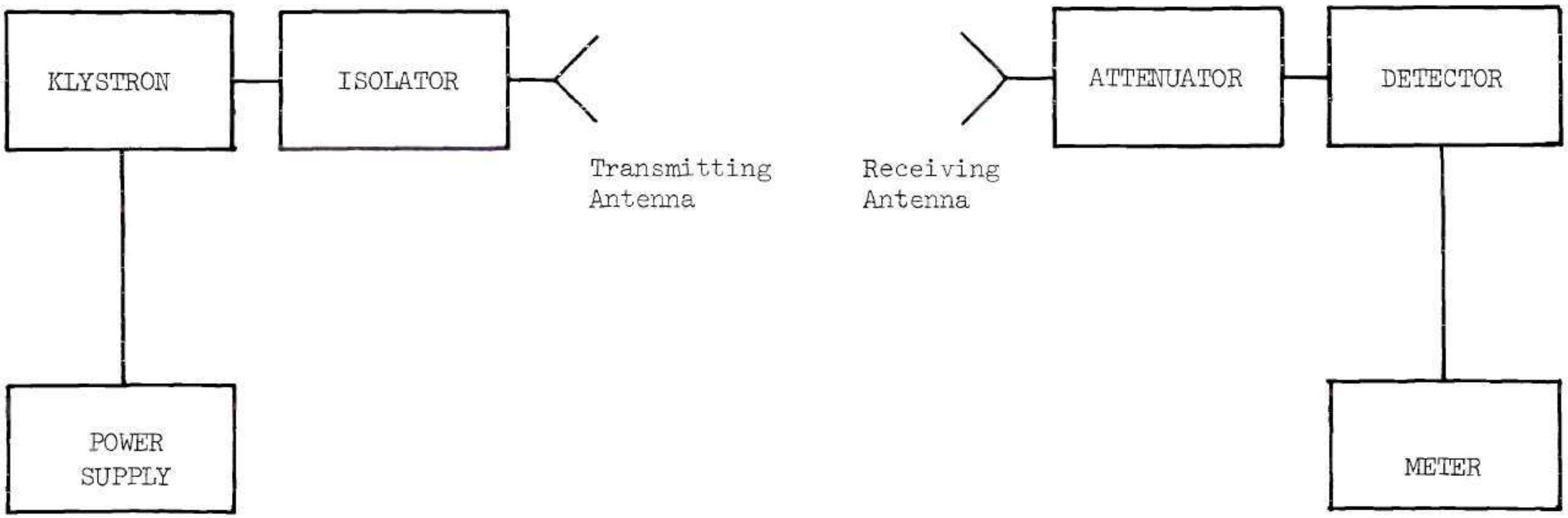


Figure 2. Block Diagram of the Experimental Set Up

polarized" as used here means that the major axis of the polarization ellipse is oriented approximately in the vertical direction.

Antenna B is now rotated about the boresight axis until maximum response is observed at the detector. At this point, the directions of principal polarization of the two antennas are coincident. After a reference level is established, antenna B is rotated again until the minimum response is observed at the detector. Were these two antennas purely linearly polarized, the minimum response would be a zero null. This would obviously occur when the directions of the two linear polarizations were exactly orthogonal. In general, however, the two antennas are elliptically polarized, and, in accordance with the mathematical analysis, one does not expect a zero null unless the two have conjugate characteristics.

The next step is to make the antennas conjugates of each other by changing the characteristics of one or both of them until a zero null is obtained. This is accomplished by distorting the waveguide input section with pressure applied through the clamp provided for this purpose. The clamp pressure is varied until minimum response is observed at the detector. The antenna is then slightly rotated about the boresight until a new smaller minimum is obtained. These two operations are repeated alternately until the minimum response is down to the noise level of the measuring system, which for all practical purposes represents a zero null.

After the two antennas have been successfully nulled against each other, antenna B is replaced by the third antenna. The same nulling procedure is repeated with antennas A and C with one important exception. The characteristics of antenna A must not be changed at this time. In other words, the entire nulling process must be accomplished by adjusting antenna C alone. The characteristics of antenna A must be exactly the same as they were when A was nulled against B since antennas B and C must have identical polarization characteristics. Since, after the nulling procedure, antennas B and C are both conjugate to A, it is obvious that the characteristics of A must not be changed between the two nulling procedures.

With the completion of this first step in the procedure, antennas B and C have identical polarization characteristics. They are, in general, elliptically polarized, and their respective polarization ellipses have the same ratio of major to minor axes and the same sense of rotation. In addition, the polarization characteristics of antenna A are conjugate to the characteristics of both antenna B and antenna C.

It is now necessary to set up antennas B and C as transmitter and receiver. In the interest of definiteness and simplicity, antenna C will be left in position as the receiver, and antenna B will replace antenna A as the transmitter. The boresight axes of the respective antennas are aligned as before. There now exists a system coupling power between two antennas which have identical polarization characteristics. Either of the two antennas is now rotated about the boresight axis, and the ratio of maximum response to minimum response is measured and recorded. If this ratio is infinite, meaning that the minimum response is a zero null, then the work is complete. This follows from the fact that these two antennas have identical polarization characteristics and, as discussed in Chapter II, a zero null between two such antennas is possible if and only if both antennas are purely linearly polarized. In general, however, the ratio will have some finite value which simply means that both antennas are elliptically polarized. Consequently, to achieve a zero null it will usually be necessary to change the characteristics of both antennas. Quite obviously, the characteristics of both antennas must be changed by the same amount in order to preserve their identity. The difficulty of performing an identical change in the two antennas is readily appreciated from an experimental point of view. An alternative approach is thus desirable.

The characteristics of antenna A, the original comparison antenna, can be slightly varied by either increasing or decreasing the clamp pressure on antenna A. Note should be made of whether the pressure is increased or decreased. Antenna A, with its characteristics thus changed, is introduced back into the experimental system. The entire experimental procedure is repeated. When antennas B and C are again compared, the ratio of maximum to minimum response will take on a larger or smaller value than that previously obtained. As mentioned above, it is desirable to have this ratio approach infinity. Hence, if the new ratio is greater, the change in clamp pressure on antenna A was made in the right direction. This trial and error procedure is repeated, until the largest possible ratio is obtained. The larger the ratio, the more nearly pure is the resulting linear standard. Also, since antenna A is the conjugate of the two test antennas, it too is linearly polarized and the objective has been achieved.

A good check of the final result is to make certain that any two of the three antennas chosen at random can be nulled against each other by a simple rotation about the boresight axis. If this can be done, all three of the antennas are linearly polarized.

Orientation of the Direction of Polarization

The first objective having been accomplished, it is now necessary to determine a method of orienting the linearly polarized field along any arbitrary space axis. Conventionally, if a vertically polarized field is desired, one usually places a bubble level across the broad dimension of the radiating guide and levels this surface with the earth. However, construction techniques make it quite difficult, if not impossible, to define such a surface which is exactly perpendicular to the radiated field. Consequently, it is necessary to devise a means of defining a surface or line on the physical antenna which is perpendicular to or parallel to the radiated field polarization direction. This surface or line may then be oriented in the desired direction with an accuracy limited only by the leveling device used. This objective too can be accomplished by a comparison technique.

Select two of the three linearly polarized antennas, and orient them as before with one operating as a transmitter, and the other operating as a receiver. For the sake of definiteness, let antenna A be the transmitter, and antenna B be the receiver. Define a rectangular coordinate system such that the X-axis is vertical, the Z-axis is coincident with the boresight of the two antennas, and the Y-axis is horizontal. Now level the appropriate side of the transmitting waveguide such that this direction of polarization may be assumed to be vertical. As noted above, the direction of polarization will not be exactly vertical but will differ from coincidence with the X-axis by some angle α as shown in Figure 3.

If α can be determined, the true direction of polarization will

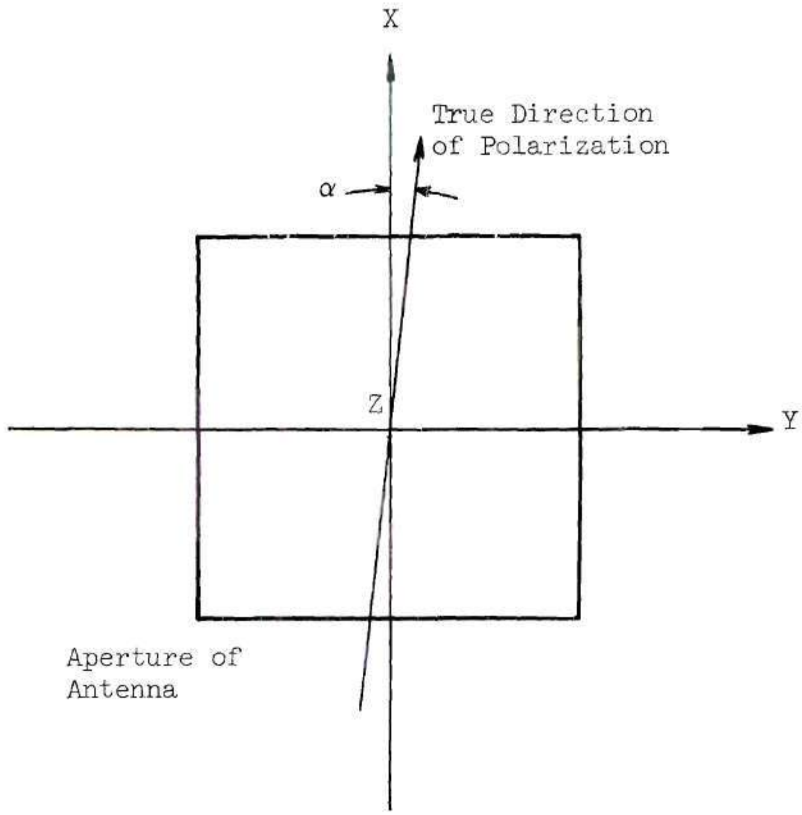


Figure 3. Diagram Showing the Relationship Between the Aperture, the Coordinate System, and the True Direction of Polarization.

be known. If antenna B is now rotated about the boresight axis to a null, then the direction of polarization of antenna B is exactly orthogonal to that of antenna A; therefore, the direction of polarization of B differs from coincidence with the Y-axis by the same angle α. A convenient surface is now chosen on antenna B, and the angle this surface makes with the horizontal is measured and recorded. Denote this angle by β.

The same procedure is now repeated with antenna B replaced by antenna C. Since antenna A is unchanged, the direction of polarization of antenna C, after nulling, differs from horizontal by the same angle α. A convenient surface is chosen on antenna C and its angle with respect to horizontal is measured and recorded. Denote this angle by γ. Due to construction differences in the three antennas, this angle is in general not equal to the angle previously measured with antenna B. The situation is depicted in Figure 4 which is a diagram of the two antennas as seen looking down the positive Z-axis.

Antenna A is now replaced by antenna B as the transmitter, and antenna B is oriented in the identical angle in which it was previously oriented. However, the boresight of antenna B now coincides with the positive Z-axis whereas when used as a receiver the boresight coincided with the negative Z-axis. As far as the coordinate system is concerned, changing antenna B from a receiver to a transmitter amounted to a 180° rotation of the antenna about the X-axis. Figure 5 is a diagram of the two antennas in which the aperture of antenna B, the transmitter, is as seen by an observer standing behind it and looking in the direction of propagation. Antenna C is as seen by an observer looking into the

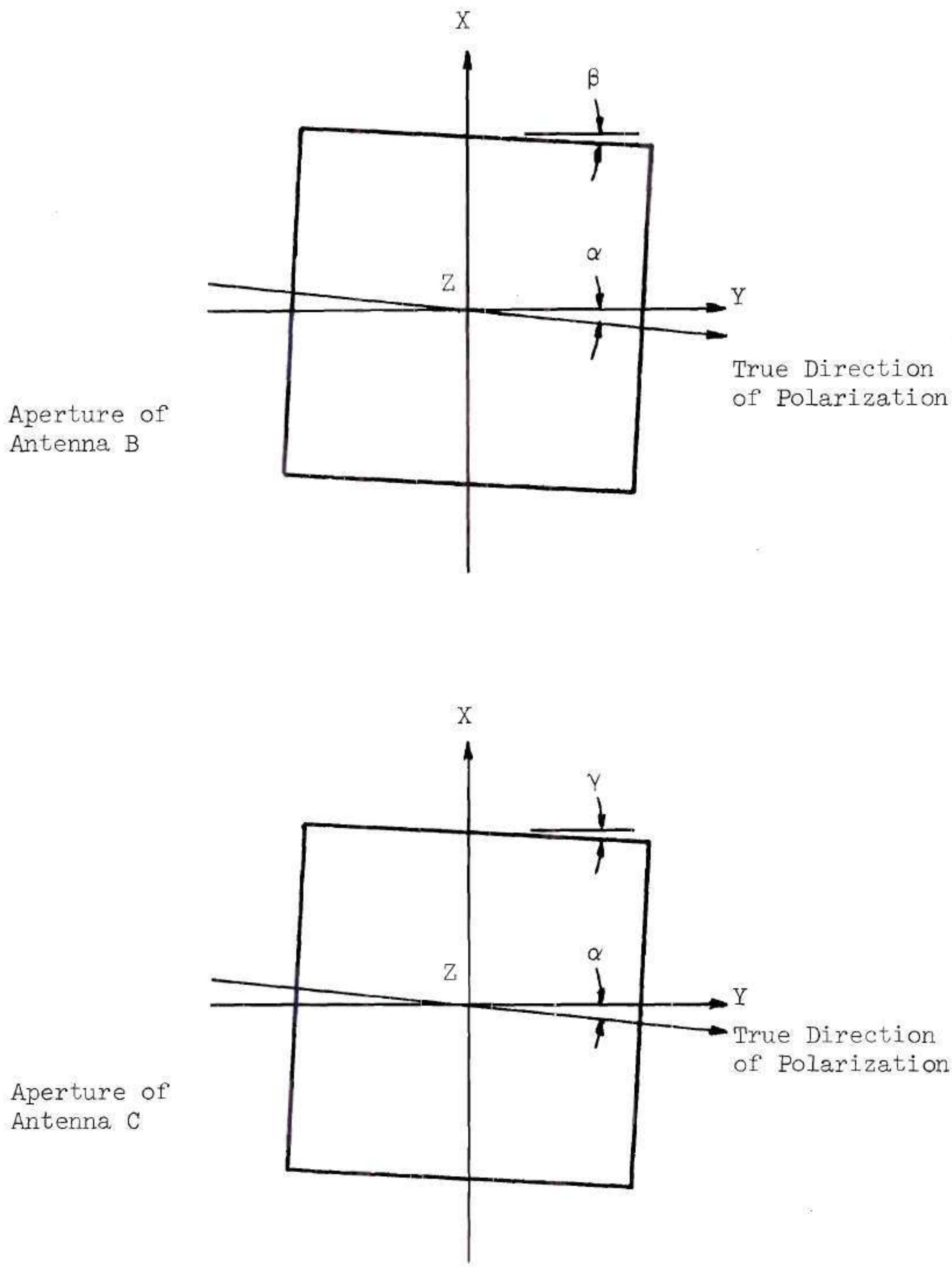
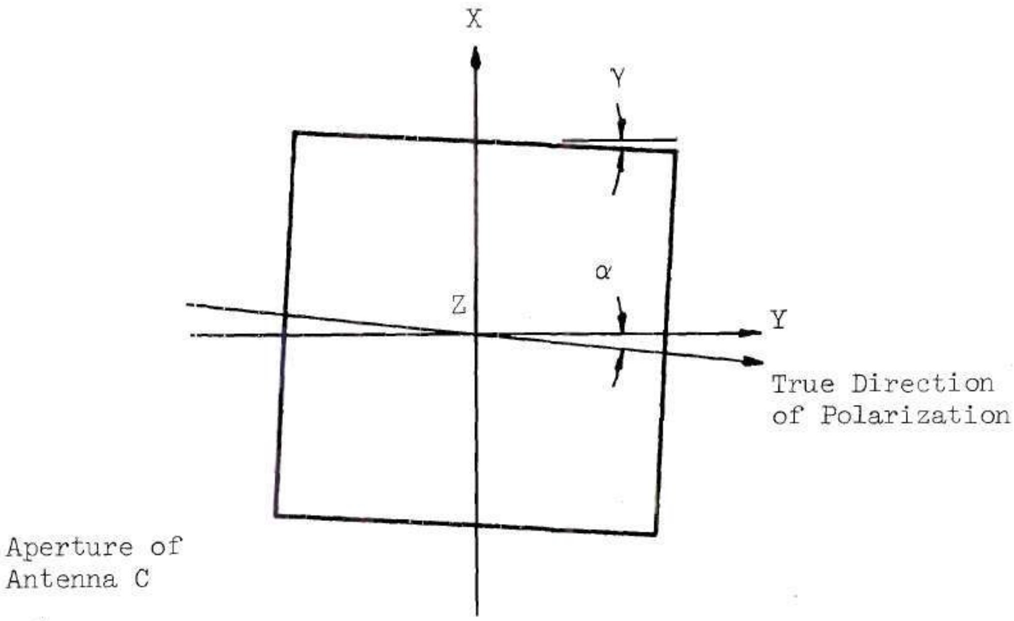


Figure 4. Diagram Showing the Relationship Between the Apertures, the Coordinate System, and the True Direction of Polarization of Antennas B and C After Each has Been Nulled Against A. The Angles β and γ Are Also Shown.



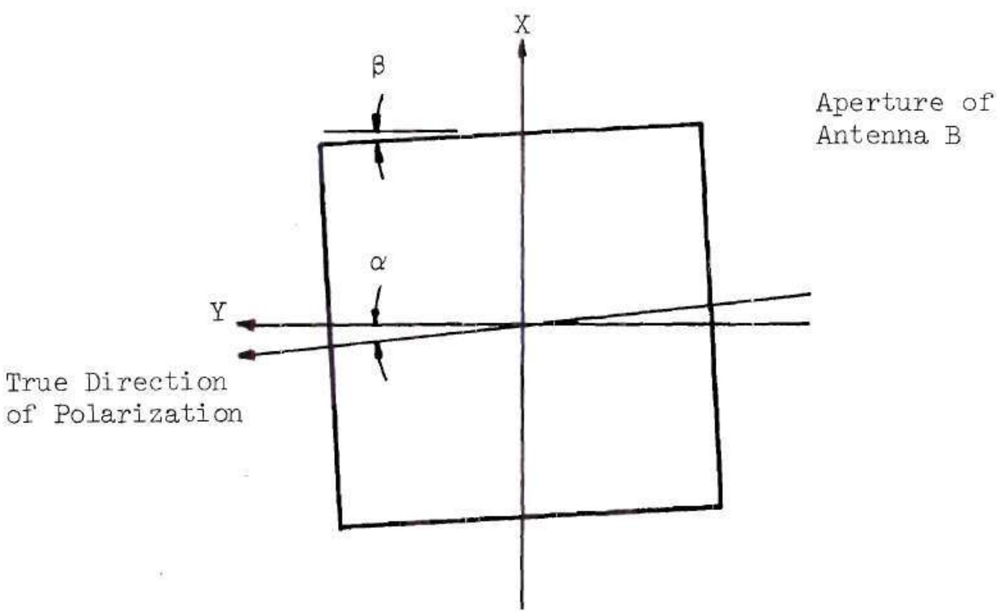


Figure 5. Diagram Showing the Orientation of the True Direction of Polarization After Antenna B Has Been Rotated 180 Degrees About the Y-Axis.

aperture in the direction of propagation. Note the significant difference in Figures 4 and 5 caused by the 180° rotation of antenna B about the X-axis. The actual directions of polarization of the two antennas differ by the amount 2α. Inspection of Figure 5 shows that a null can be obtained between these two antennas only by rotating either of the two antennas by $90^\circ \pm 2\alpha$ depending on the direction of rotation. One only needs to measure the angle of rotation required to obtain a null and equate this number to $90^\circ \pm 2\alpha$. As pointed out above, once α is known, then the true direction of polarization is known.

CHAPTER V

RESULTS

The results of the experimental phase of the research were quite consistent with the results predicted on the basis of the mathematical theory. It was found that cross components of polarization could be reduced from about 35 db or 40 db to as much as 65 db or 70 db below the parallel component. As expected, the most difficult part of the experiment was the trial and error procedure of obtaining the zero null between the two identical antennas. This required changing the polarization characteristics of each antenna by an equal amount which was quite difficult because of the method used. Consequently, after each change, the two antennas had to be compared with the third antenna to insure that the identity of their polarization characteristics had been maintained. It was found that an average of four to six repeat comparisons were required before a satisfactory null was obtained.

The phase of the experiment designed to orient the true direction of polarization revealed that the conventional method of leveling the broad dimension of the waveguide to obtain vertical polarization produced errors on the order of one to two degrees. These errors created as much as 20 db difference in the level of the null; the importance of reliable positioning of the antenna thus became evident. An example of the typical numerical results achieved is presented in the Appendix in the form of a sample data sheet. Some specific aspects of the experiment are discussed in more detail below.

Adjustment of Polarization Characteristics

As mentioned earlier, the technique for adjusting the polarization characteristics was to distort the square waveguide input section by applying a clamping pressure across the diagonal of the section. Quite obviously, this method is not completely satisfactory since the amount of change in polarization characteristics produced by a given increase in pressure is not easily controlled and is not necessarily repeatable. In addition, the fact that the same change in pressure on two antennas will not, in general, produce the same change in their respective polarization characteristics necessitates the difficult trial and error procedure, and a considerable improvement in the procedure would be realized if a method of calibrating the change in polarization characteristics were available.

System Noise Level

The major limitation to the accuracy of the method is caused by the noise level of the measurement system. Consideration of the procedure reveals that the success of the method relies on the ability to measure a zero null. Obviously, the more accurate the null measurement, the more accurate will be the final calibration.

The noise level of the system used in the research was found to be approximately 70 db below the level of maximum power transfer between the two antennas. Consequently, the minimum value of cross component obtainable with this system obviously cannot be less than 70 db below the primary component.

Range Effects

It is well known that the reliability of virtually all antenna measurements is strongly dependent on the quality of the antenna range used for the measurements; this is particularly true in the case of low level measurements such as those encountered in this experiment. Nearly all antenna ranges have features which cause the creation of secondary sources of radiation through the processes of reflection and diffraction of the range illumination radiation. These secondary sources of radiation cause undesirable interference patterns in the energy incident on the test antenna. Quite obviously, the level of these secondary sources of radiation must be kept negligibly low in order to assure confidence in the measurements being made. What constitutes a negligibly low level of secondary radiation depends largely on the desired dynamic range of the measurement. For example, if one wishes to measure an antenna radiation pattern over a dynamic range of 40 db, it is necessary to maintain the level of secondary sources much less than 40 db below the direct radiation.

The measurements made in this research require a dynamic range of approximately 65 db to 70 db. Unfortunately, the level of secondary sources of radiation in the microwave dark room used for this work is not believed to be compatible with this requirement (7). Consequently, the actual calibration of the horns is not believed to be as good as the data reveal. However, the primary objective of this research is the demonstration of the method rather than the actual calibration. The demonstration of the method is unaffected by the range if the horns are always used in the same positions in the room. This follows from the fact that, at a given point, all the components of radiation can be combined into a single elliptically polarized wave by vector and phasor addition. The resulting restriction on the calibration is that it applies only in the dark room when the horns are in the same positions as they were when calibrated.

CHAPTER VI

CONCLUSIONS

It has been demonstrated that a method exists for calibrating a linear polarization standard antenna. Furthermore, the feasibility of the method has been verified by experimentally carrying out the procedure. It was found that, as a result primarily of the range effects, the actual calibration achieved was not as good as could be expected if performed on a better antenna range. However, the experiment did confirm that the method is feasible.

If the method were applied on a good antenna range, it is estimated that cross components of polarization in a supposedly linearly polarized field can be reduced to levels of 65 to 70 db below the primary components. The availability of such an antenna will enable polarization measurements on antennas with an accuracy thus far unobtainable with conventional standards.

CHAPTER VIII

RECOMMENDATIONS

Although the calibration method presented here is thought to be satisfactory, there is some room for improvement in the actual mechanics of the calibration.

Less time would be required for performing the calibration if a method of varying the polarization characteristics of the test antennas was available which would allow changing the characteristics of the antennas by a known amount. This would allow elimination of the time-consuming trial and error procedure of obtaining a null between two antennas and at the same time would maintain the identity of their polarization characteristics.

Care should be taken to insure that the antenna range and the noise level of the measuring system are compatible. That is, both secondary reflections from the range and the noise level of the measuring system should be kept as low as possible. However, if the best available range has reflections at -50 db, there is no need to obtain a measuring system with a noise level less than -50 db. Conversely, if the best available measuring system has a noise level at -50 db, then there is no need to find a range with reflections less than -50 db. For most practical applications, it appears that both antenna range reflections and measuring system noise level should be maintained at a level of about -70 db.

During the course of the experiment, it was observed that the calibration is quite sensitive to any changes in the horns or the clamp pressure. Consequently, it is felt that the calibration may not be very stable as a function of usage, temperature changes, or other effects which tend to distort the surfaces of the antenna. It is therefore recommended that considerable attention be directed toward the mechanical characteristics of the antenna to be used for the calibration.

APPENDIX A

MATHEMATICAL DERIVATIONS

The pertinent mathematics of power transfer between two arbitrarily polarized antennas are presented here. The analysis is based on the work of Sinclair (8) and Olin (9).

The electric field at a point remote from a transmitting antenna may be written in terms of a rectangular coordinate system:

$$\overline{E}^t = \overline{u}_x E_x e^{j\omega t} + \overline{u}_y E_y e^{j(\omega t + \delta_y)} + \overline{u}_z E_z e^{j(\omega t + \delta_z)} \tag{1}$$

where the propagation phase factor has been suppressed and the phase of each component is referenced to the phase of the X component.

We can arbitrarily choose the Z-axis to coincide with the direction of propagation. Then, since in the far zone the electric field is transverse to the direction of propagation, $E_z = 0$. Equation (1) becomes:

$$\overline{E}^t = \overline{u}_x E_x e^{j\omega t} + \overline{u}_y E_y e^{j(\omega t + \delta_y)}. \tag{2}$$

Equation (2) may be written as:

$$\overline{E}^t = E_o \overline{h} \tag{3}$$

where E_o is a positive real amplitude constant and,

$$\overline{h} = \overline{u}_x M e^{j\omega t} + \overline{u}_y N e^{j(\omega t + \delta_y)} \tag{4}$$

$$= \overline{u}_x h_x + \overline{u}_y h_y$$

is a complex vector called the "vector height" (8) of the antenna. In Equation (4), M and N are positive real numbers. Note that the complex vector $\overline{h}$ describes the polarization characteristics along the boresight axis. For purposes of this analysis, we shall restrict our attention to points of observation on the boresight axis of the antenna. Hence the aperture will be contained in the XY plane of the chosen coordinate system. In addition, we shall choose the X-axis to be vertical and the Y-axis horizontal. Hence, h_x describes a vertically polarized component, and h_y describes a horizontally polarized component.

When used as a receiver, the induced open-circuit voltage at the terminals of the antenna is given by

$$V = \overline{h} \cdot \overline{E}^r \tag{5}$$

where $\overline{E}^r$ is the incident or received field. (8)

Consider now two antennas, designated A and B, oriented in a transmission system with their respective boresight axes coincident. Figure 6 illustrates the two antennas and their respective coordinate systems when used as transmitters and thus for definition of $\overline{h}$. The vector height is given for each antenna as:

$$\overline{h}^A = \overline{u}_x^A h_x^A + \overline{u}_y^A h_y^A \,, \tag{6a}$$

$$\overline{h}^B = \overline{u}_x^B h_x^B + \overline{u}_y^B h_y^B \,. \tag{6b}$$

If antenna A is the transmitter, the transmitted field is from Equation (3)

$$\overline{E}^t = E_o \overline{h}^A \,. \tag{7}$$

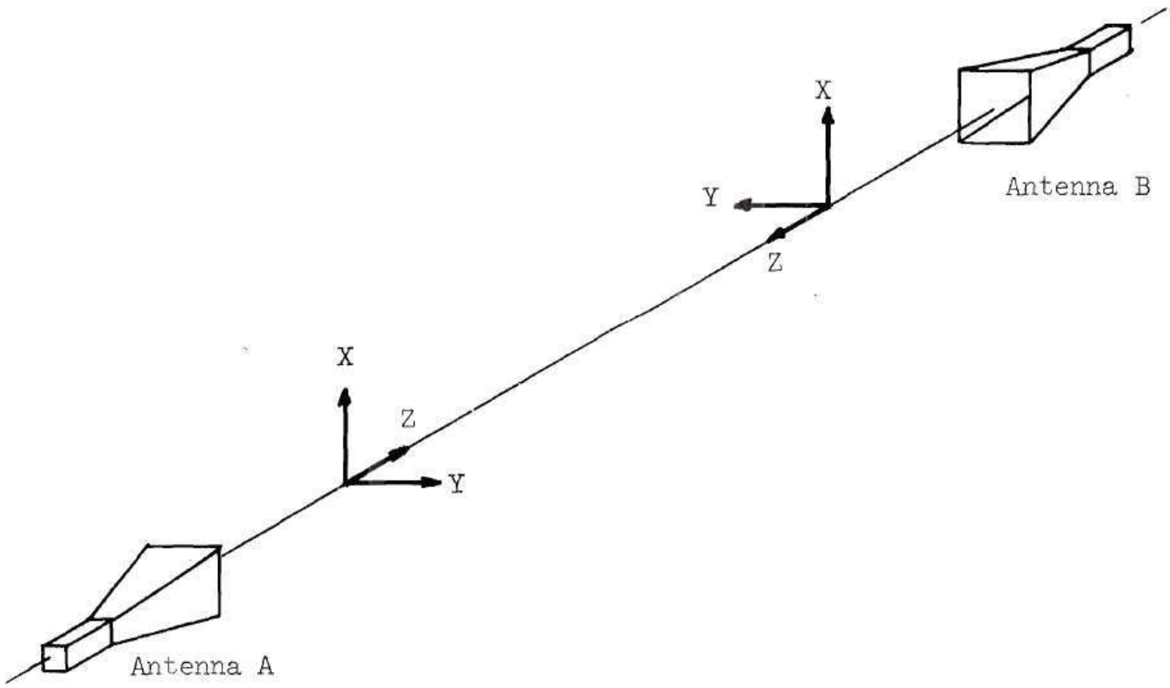


Figure 6. Diagram Showing the Two Antennas and Their Respective Coordinate Systems for Definition of $\overline{h}$.

Since the field transmitted by antenna A is received by antenna B

$$\overline{E} = E_o(\overline{u}^A_x h^A_x + \overline{u}^A_y h^A_y) = E_o(\overline{u}^B_x h^B_x + \overline{u}^B_y h^B_y) \ . \tag{8}$$

From Figure 6

$$\overline{u}^A_x = \overline{u}^B_x \ , \tag{9a}$$

$$\overline{u}^A_y = -\overline{u}^B_y \ . \tag{9b}$$

Hence, Equation (8) may be written:

$$\overline{E} = E_o(\overline{u}^B_x h^A_x - \overline{u}^B_y h^A_y) = E_o(\overline{u}^B_x h^B_x + \overline{u}^B_y h^B_y) \ . \tag{10}$$

Therefore, with respect to the receiver coordinate system, the field received by antenna B is

$$\overline{E}^r = E_o(\overline{u}^B_x h^A_x - \overline{u}^B_y h^A_y) \ . \tag{11}$$

From Equation (5)

$$V^B = \overline{h}^B \cdot \overline{E}^r$$

$$= E_o(h^A_x h^B_x - h^A_y h^B_y) \ . \tag{12}$$

Substituting in Equation (12) the values for h^A_x, h^B_x, h^A_y, and h^B_y given by Equation (4):

$$V^B = E_o \left[M_A M_B e^{j2\omega t} - N_A N_B e^{j2\omega t} e^{j(\delta_A + \delta_B)} \right]$$

$$= E_o^{j2\omega t} \left[M_A M_B - N_A N_B e^{j(\delta_A + \delta_B)} \right] \ . \tag{13}$$

Equation (13) is an expression for the open circuit voltage induced at the terminals of one elliptically polarized antenna as a result of its receiving a field transmitted by a second elliptically polarized antenna.

Of particular interest here is the determination of the conditions under which no power is transmitted between the two antennas. Obviously, this occurs when V^B of Equation (13) vanishes. Hence,

$$M_A M_B - N_A N_B e^{j(\delta_A + \delta_B)} = 0 \ .$$

An obvious solution occurs when

$$M_A = N_B = 0$$

or

$$M_B = N_A = 0 \ .$$

Inspection of Equation (4) shows that this solution corresponds to two linearly polarized antennas whose directions of polarization are orthogonal. A second solution occurs when

$$M_A M_B = N_A N_B \ ,$$

$$\delta_B = -\delta_A \ .$$

From which,

$$\frac{M_A}{N_A} = \frac{N_B}{M_B} \ ,$$

$$\delta_B = -\delta_A \ .$$

This solution corresponds to two elliptically polarized fields whose polarization ellipses described with respect to the same coordinate system have inverse ratios of x to y components and opposite senses of rotation. By definition, the two antennas under these conditions are said to have conjugate polarization characteristics. Figure 7 shows a possible orientation of the field components of the two antennas and the phase angle δ for a condition of no power transfer.

The important conclusion to be derived from these results is that a zero null of power transfer between two arbitrarily polarized antennas is possible only when the fields of the two antennas are linearly polarized, or when the polarization characteristics are conjugate to each other in accordance with the above definition. Consequently, when two antennas with identical polarization characteristics can be nulled out, they must necessarily both be linearly polarized.

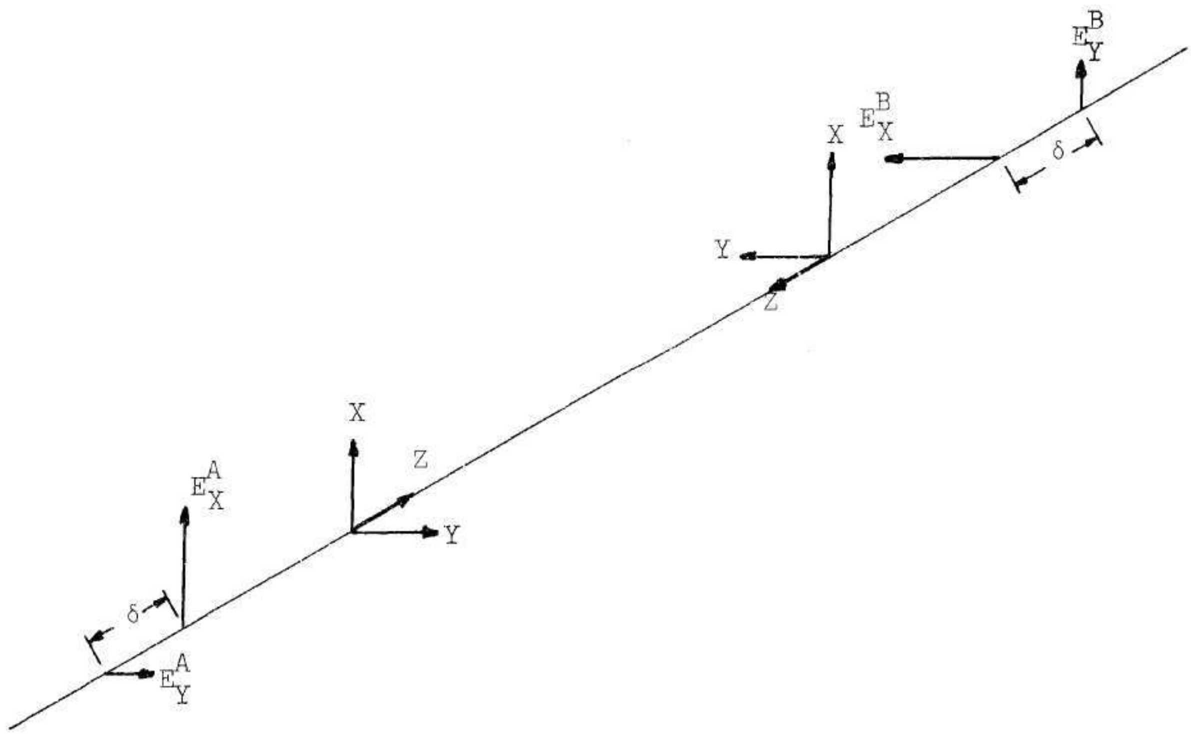


Figure 7. A Diagram Showing a Possible Orientation of the Field Components of the Two Antennas and the Phase Angle δ for a Condition of No Power Transfer.

APPENDIX B

SAMPLE DATA SHEET

PART I

Comparison No.	Ratio (db)	Clamp Pressure
1	35	______
2	32	Decrease
3	40	Increase
4	53	Increase
5	60	Increase
6	>65	Increase

PART II

Measured Angles

β $\underline{\quad 1^\circ \quad}$

γ $\underline{\quad 1.5^\circ \quad}$

Calculated Angle

α $\underline{\quad 1^\circ \quad}$

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